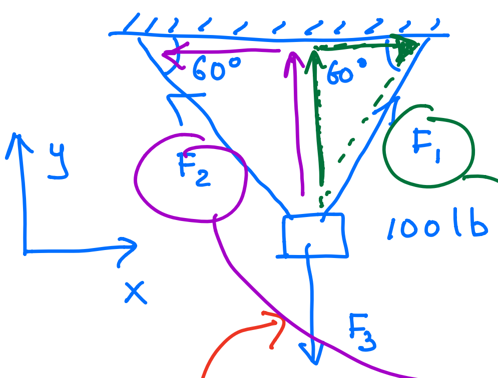
Problem:

If 100 lb load is suspended by two chains as in pic.
what is the magnitude of force each chain must be able to support.



Need to find $|F_1|$, and $|F_2|$

$$F_1 + F_2 + F_3 = (0,0)$$

$$F_1 = (|F_1| \cdot \cos 60, \ |F_1| \cdot \sin 60^\circ)$$

$$F_2 = (-|F_2| \cos 60, \ |F_2| \cdot \sin 60^\circ)$$

$$F_3 = (0, -100)$$

$$F_1+F_2+F_3 = (|F_1|\cos 60 - |F_2|\cos 60, \ |F_1|\sin 60 + |F_2|\sin 60 - 100) = (0,0)$$

$$|F_1|\cos 60 - |F_2|\cos 60 = 0$$

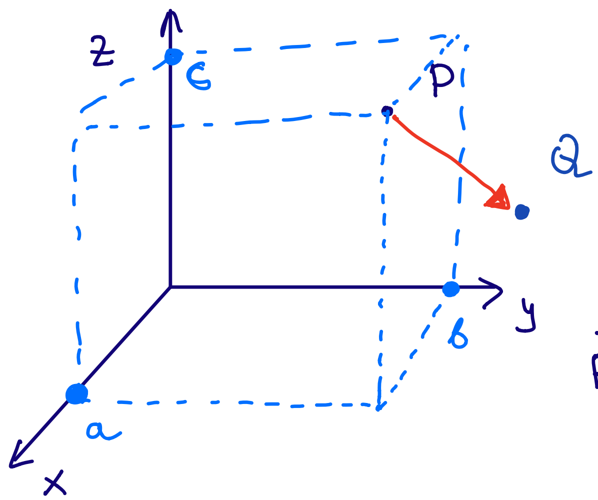
$$|F_1| - |F_2| = 0 \Rightarrow |F_1| = |F_2|$$

$$|F_1|\sin 60 + |F_2|\sin 60 - 100 = 0$$

$$2|F_1| \cdot \sin 60 = 100 \Rightarrow |F_1| = \frac{100}{2 \cdot \sin 60} = \frac{50}{(\sqrt{3}/2)}$$

$$\boxed{|F_1| = |F_2| = \frac{100}{\sqrt{3}}}$$

Vectors in 3 D:



$P = (a, b, c)$

$Q = (a', b', c')$

$\overrightarrow{PQ} = Q - P = (a'-a, b'-b, c'-c)$

We cand add, subtract vectors in 3D and multiply them by numbers:

Ex: $v = (\alpha, \beta, \gamma) \qquad u = (x, y, z)$

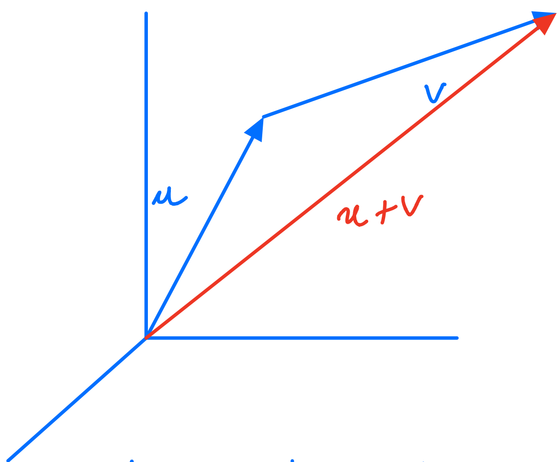
$$v + u = (\alpha + x, \beta + y, \gamma + z)$$

$$c \in \mathbb{R}$$

$$c \cdot v = (\alpha \cdot c, \beta \cdot c, \gamma \cdot c)$$

$$v + c \cdot u = (\underbrace{\alpha + x \cdot c}_{\alpha + c \cdot x}, \beta + y \cdot c, \gamma + z \cdot c)$$

in components



in picture

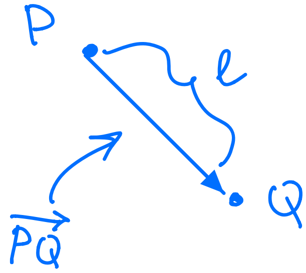
Magnitude of vector:

$$v = (v_x, v_y, v_z)$$ ← vector in 3D

$v_x, v_y, v_z \in \mathbb{R}$

$$\boxed{|v| = \sqrt{v_x^2 + v_y^2 + v_z^2}}$$

Example: Find distance between points $P = (1,1,1)$ and $Q = (0,0,2)$



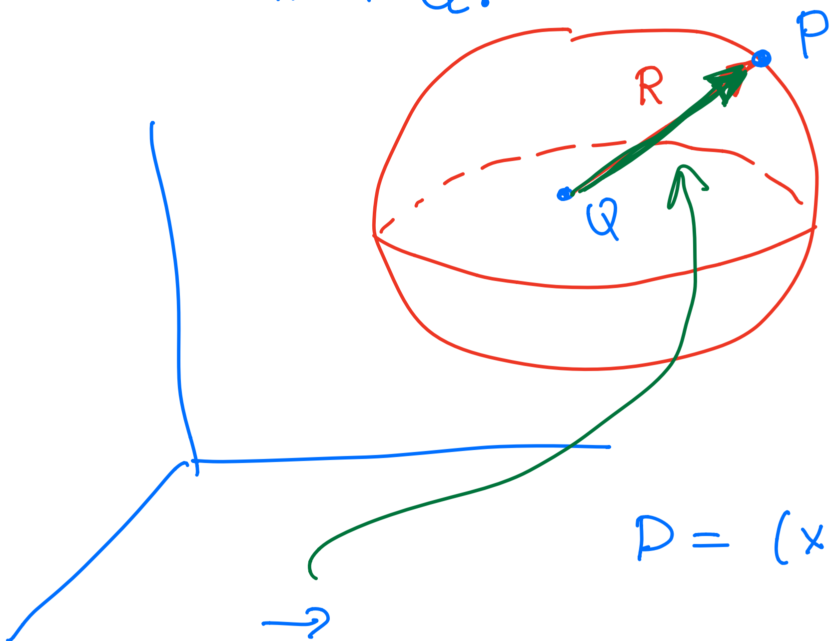
$$\vec{PQ} = Q - P = (0,0,2) - (1,1,1)$$

$$= (-1, -1, 1)$$

$$|\vec{PQ}| = \sqrt{(-1)^2 + (-1)^2 + (1)^2} = \sqrt{3}$$

Example: Assume $Q = (a, b, c)$ is a point in 3D.

find all points (x, y, z) in ~~ℝ~~ 3D which are at the distane R from Q.



A sphere centered at Q with Radius R.

$$P = (x, y, z)$$

$$\overrightarrow{QP} = P - Q = (x, y, z) - (a, b, c)$$

$$= (x-a,\ y-b,\ z-c)$$

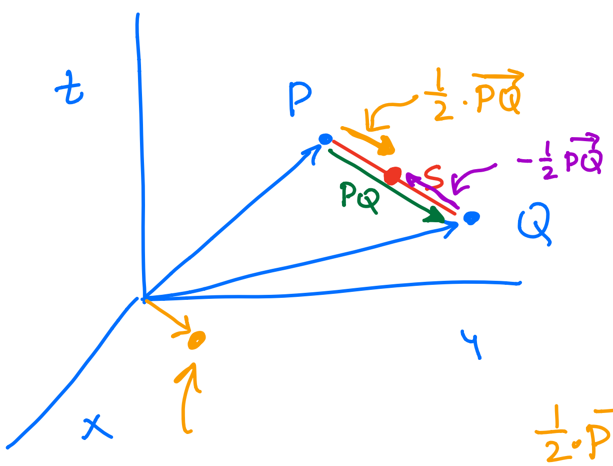
$|\overrightarrow{QP}| = R$ same as $\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = R$

$$\Rightarrow \boxed{(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2}$$

(Equation of sphere with center (a, b, c) with Radius R.

Example:

Assume $P=(x_1, y_1, z_1)$ and $Q=(x_2, y_2, z_2)$ – two points in 3D. Find the coordinates of midpoint between P, and Q.



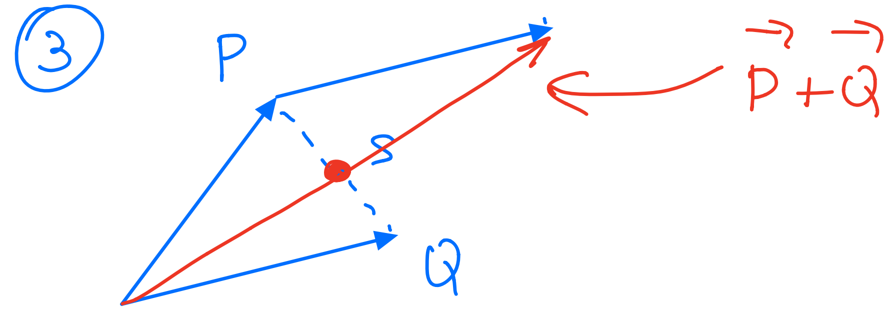
$$\overrightarrow{PQ} = Q - P =$$
$$= (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$\frac{1}{2}\cdot\overrightarrow{PQ} = \left(\frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2}, \frac{z_2 - z_1}{2}\right)$$

① $$S = P + \frac{1}{2}\overrightarrow{PQ} = (x_1, y_1, z_1) + \left(\frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2}, \frac{z_2 - z_1}{2}\right)$$
$$= \left(x_1 + \frac{x_2 - x_1}{2}, y_1 + \frac{y_2 - y_1}{2}, z_1 + \frac{z_2 - z_1}{2}\right)$$
$$= \boxed{\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)}$$

② $$S = Q - \frac{1}{2}\overrightarrow{PQ} = (x_2, y_2, z_2) - \frac{1}{2}\cdot\left(\frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2}, \frac{z_2 - z_1}{2}\right)$$
$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

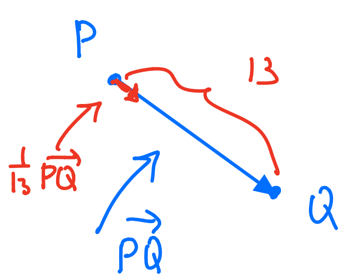
③



$$S = \frac{\vec{P}+\vec{Q}}{2} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

Unit vector in 3D = a vector with magnitude = 1

Ex: Find a unit vector in direction $\vec{PQ}$ with $P = (5,3,1)$
$Q = (-7,8,1)$



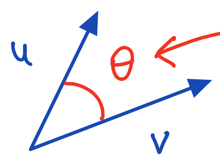
$$\vec{PQ} = Q - P = (-7-5, 8-3, 1-1)$$
$$= (-12, 5, 0)$$
$$|\vec{PQ}| = \sqrt{(-12)^2+5^2+0^2} = \sqrt{144+25}$$
$$= \sqrt{169} = 13.$$

unit vector = $\boxed{\frac{1}{13}\vec{PQ} = \left(-\frac{12}{13}, \frac{5}{13}, 0\right)}$

Dot product:

$u=(x_1,y_1)$, $v=(x_2,y_2)$

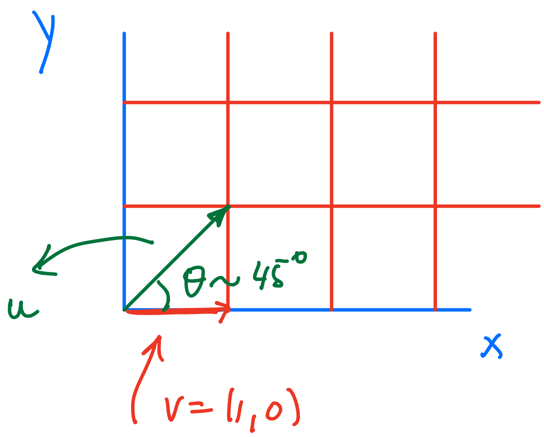
$$u\cdot v = x_1\cdot x_2 + y_1\cdot y_2 = |u|\cdot|v|\cdot\cos\theta$$



Example:

$u=(1,1)$ $v=(1,0)$

$$u\cdot v = 1\cdot 1 + 1\cdot 0 = \boxed{1}$$



$$|v| = \sqrt{1^2+0^2} = 1$$

$$|u| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\cos 45° = \frac{\sqrt{2}}{2}.$$

$$u\cdot v = |u|\cdot|v|\cdot\cos\theta$$

$$= \sqrt{2}\cdot 1\cdot \frac{\sqrt{2}}{2} = \boxed{1}$$

In 3D:

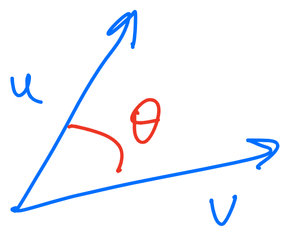
$u=(x_1,y_1,z_1)$, $v=(x_2,y_2,z_2)$

$$\boxed{u\cdot v = x_1x_2 + y_1y_2 + z_1z_1 = |u|\cdot|v|\cdot\cos\theta}$$

Example:

Find the angle between vectors

$u = (3, 4, 0)$ and $v = \langle 0, 4, 5 \rangle$



$$u \cdot v = 3 \cdot 0 + 4 \cdot 4 + 0 \cdot 5 = 16$$

$$u \cdot v = |u| \cdot |v| \cdot \cos\theta$$

$$|u| = \sqrt{3^2 + 4^2 + 0^2} = \sqrt{9 + 16} = 5$$

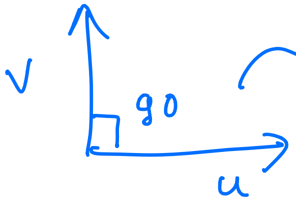
$$|v| = \sqrt{0^2 + 4^2 + 5^2} = \sqrt{16 + 25} = \sqrt{41}$$

$$u \cdot v = 5 \cdot \sqrt{41} \cdot \cos\theta$$

$$16 = 5 \cdot \sqrt{41} \cdot \cos\theta \implies$$

$$\boxed{\theta = \arccos\left(\frac{16}{5\sqrt{41}}\right)}$$

Orthogonal:



$\cos 90° = 0$

$\implies$ vectors u and v are orthogonal if

$$u \cdot v = 0$$

Example: $\begin{array}{l} V = (1,-1) \\ u = (1,1) \end{array} \Rightarrow u \cdot v = 1 \cdot (-1) + 1 \cdot 1 = 0$