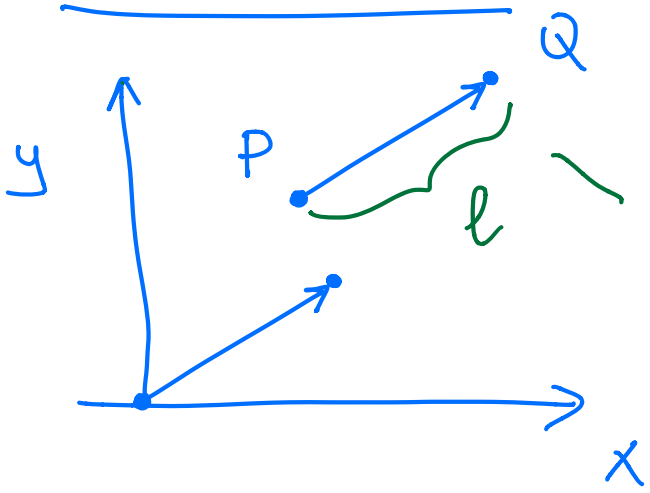
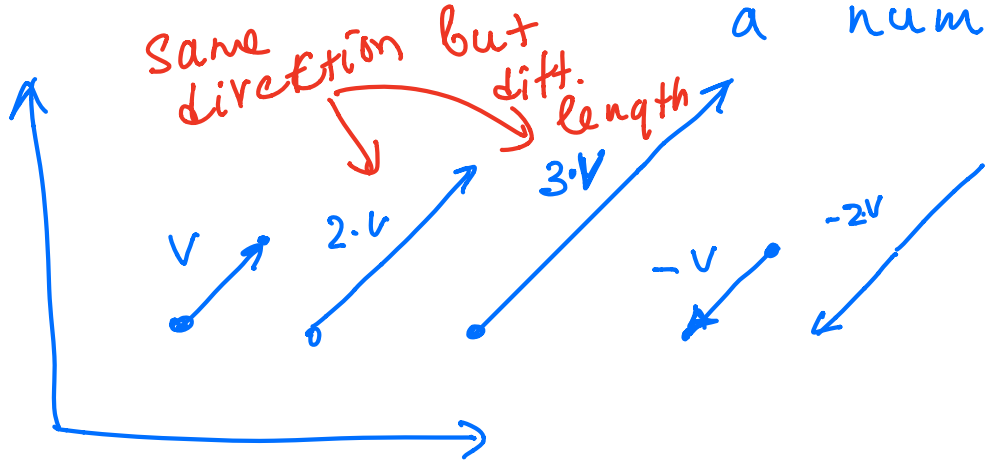
Lecture 1: (chapter 13.1)

- Vectors in $\mathbb{R}^2$
- Operations on vectors
- Components of vectors
- Magnitude of a vector
- Parallel vectors, unit vetors.
- Applications to physics.

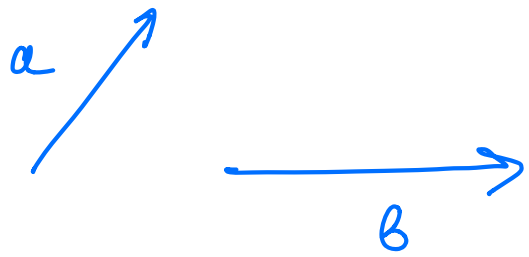
Vectors in $\mathbb{R}^2$:

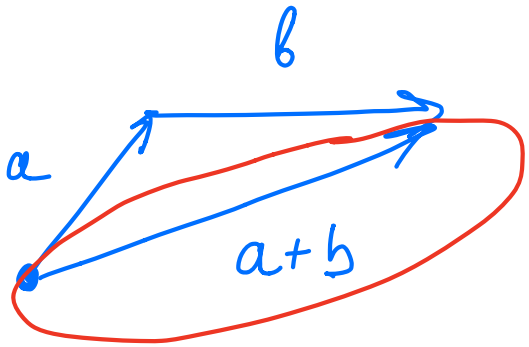


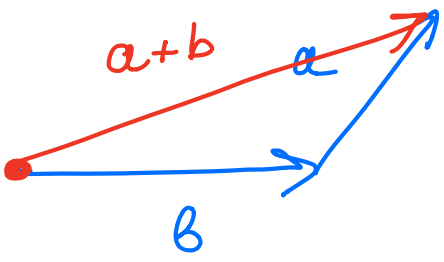
Basic operation: ① multiplication by a number $c \in \mathbb{R}$

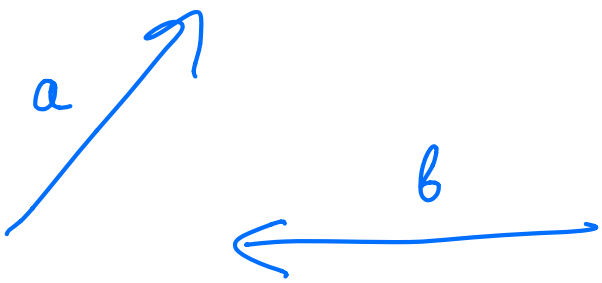


② Addition of vector :

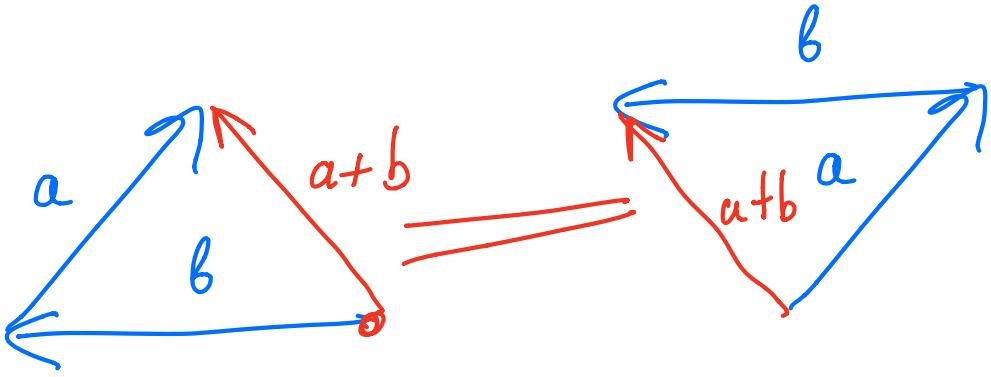




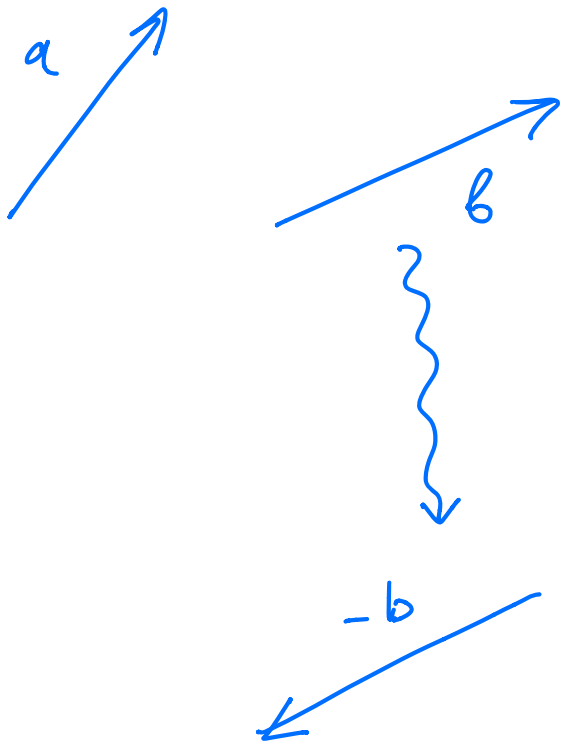




a+b :



Subtraction:



what is

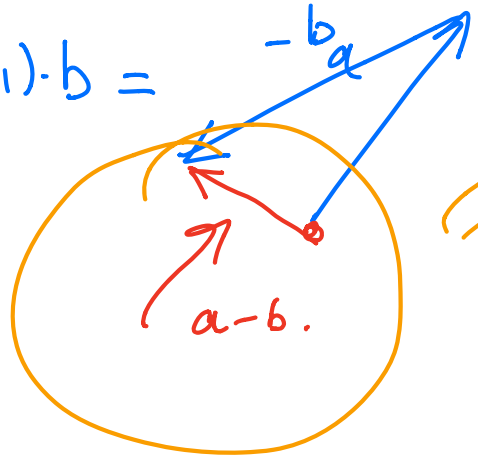
$a-b$?

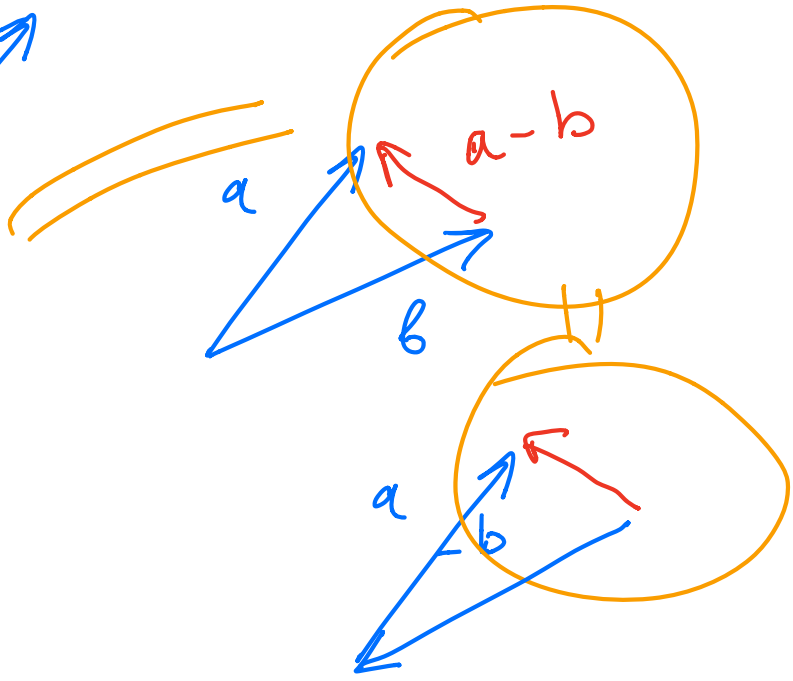
$a - b$

$=$

$a + (-1)\cdot b$

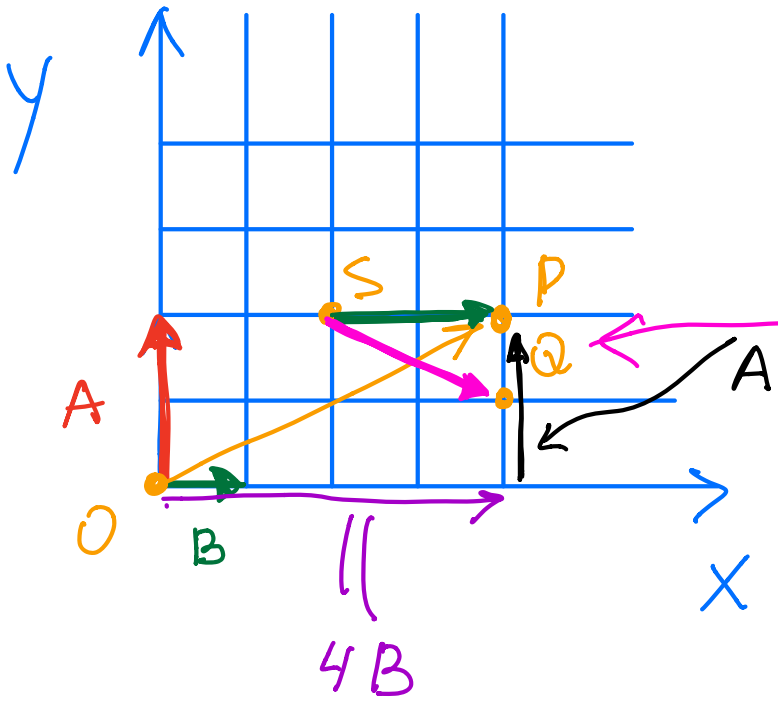
$a + (-1)\cdot b =$



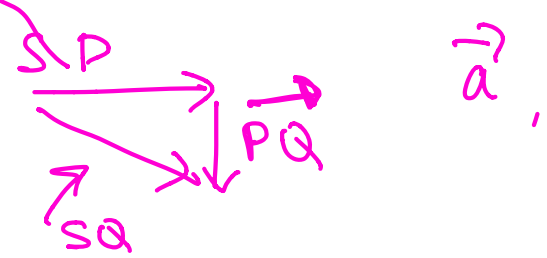


Example:

OP = in terms of A, B?



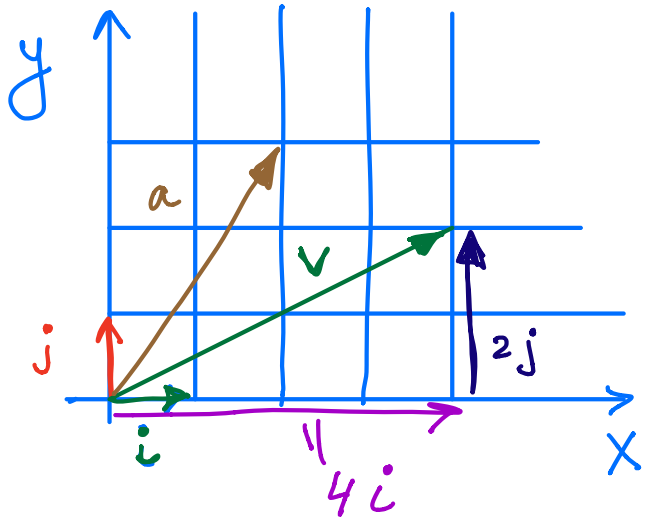
$$OP = 4B + A$$



$$SP = 2B$$

$$SQ = \underbrace{SP}_{2B} + \underbrace{PQ}_{-\frac{1}{2}A}$$

$$SQ = 2B - \frac{1}{2}A$$



j, i have legth 1.

$V = 4i + 2j$

$a = 2i + 3j$

"components" of the vector a.

Notation: $a = (2, 3)$

$V = (4, 2)$

$b = (5, 7) \iff b = 5 \cdot i + 7 \cdot j$

Addition and subtraction in componens:

$a + V = (2,3) + (4,2) = (2+4,\ 3+2) = (6,5)$

$a - V = (2,3) - (4,2) = (2-4,\ 3-2) = (-2,1)$

$$2a - 3v = 2\cdot(2,3) - 3\cdot(4,2) =$$

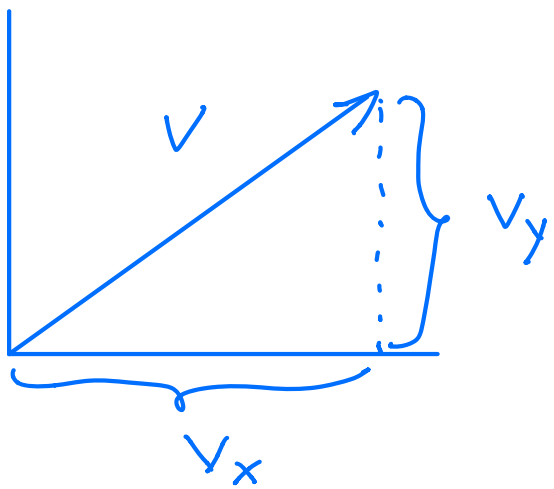
$$= (4,6) - (12,6) = (4-12, 6-6)$$

$$= (-8, 0)$$

$$\boxed{2a - 3v = (-8, 0)}$$

Length of a vector in components

$$v = (v_x, v_y)$$



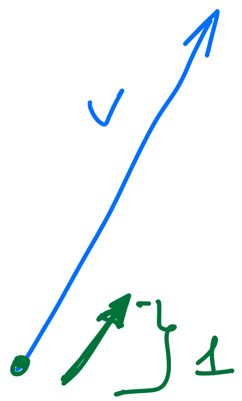
$|v|$ - length of vector v

$$\boxed{|v| = \sqrt{v_x^2 + v_y^2}}$$

Ex: $v = (2,1)$ $\quad |v| = \sqrt{2^2+1^2} = \sqrt{5}$

Ex: $V = (3,7)$

Find a unit vector (vector of length 1.) which has same direction as v.



$$|v| = \sqrt{3^2+7^2} = \sqrt{9+49} = \sqrt{58}$$

$$\frac{1}{|v|} v = \left(\frac{3}{\sqrt{58}}, \frac{7}{\sqrt{58}} \right)$$

Ex: Find b such that $v = \langle 1, b \rangle$ is a unit vector.

Solution:

$$|v| = 1$$
$$\sqrt{1+b^2}$$

$$\sqrt{1+b^2} = 1$$

$$\left(\sqrt{1+b^2}\right)^2 = 1^2$$

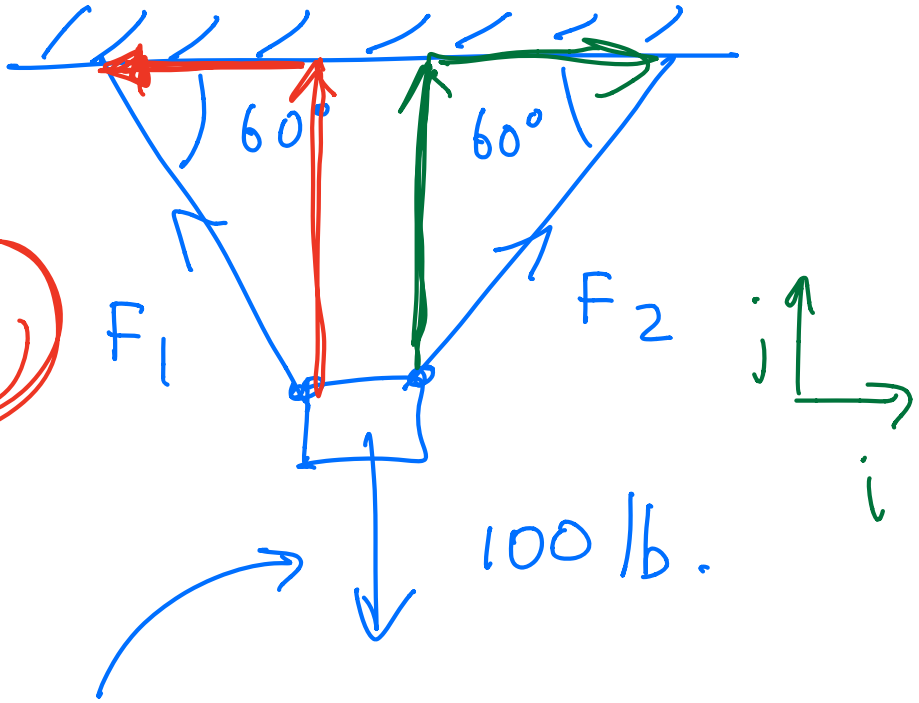
$$1+b^2 = 1$$

$$1+b^2 = 1$$

$$b^2 = 0$$

$$b = 0$$

Ex:



$|F_1| = |F_2| = S$?

W $\qquad W = (0, -100)$

$F_1 + F_2 + W = (0, 0)$

$F_1 = (-|F_1| \cdot \cos 60°, \; |F_1| \cdot \sin 60°)$ $\quad (|F_1| = S)$

$F_2 = (|F_2| \cdot \cos 60, \; |F_2| \cdot \sin 60°)$ $\quad (|F_2| = S)$

$\Rightarrow (0, 2S \sin 60° - 100) = (0, 0)$

$\Rightarrow \quad 2S \sin 60° = 100$

$S = \dfrac{100}{2 \cdot \sin 60°} = \dfrac{50}{\sin 60°}$

$$= \frac{50}{\sqrt{3}/2} = \frac{100}{\sqrt{3}}$$